NAG Toolbox for MATLAB

f08gn

1 Purpose

f08gn computes all the eigenvalues and, optionally, all the eigenvectors of a complex n by n Hermitian matrix A in packed storage.

2 Syntax

```
[ap, w, z, info] = f08gn(jobz, uplo, n, ap)
```

3 Description

The Hermitian matrix A is first reduced to real tridiagonal form, using unitary similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz – string**

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo – string**

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n Hermitian matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work, rwork

5.4 Output Parameters

1: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

ap contains the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.

2: **w**$(*)$ **– double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

If **info** $= 0$, the eigenvalues in ascending order.

3: **z**(**ldz**,$*$) **– complex array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', **ldz** $\geq \max(1, \mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'V', then if **info** $= 0$, **z** contains the orthonormal eigenvectors of the matrix A, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If **jobz** = 'N', **z** is not referenced.

4: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **rwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

Each eigenvector is normalized so that the element of largest absolute value is real and positive.

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is f08ga.

9 Example

```
jobz = 'No vectors';
uplo = 'U';
n = int32(4);
ap = [complex(0, +0);
     complex(0, +0);
     complex(0, +0);
     complex(3, -1);
     complex(3, -2);
     complex(3, +0);
     complex(4, -1);
     complex(4, -2);
     complex(4, -3);
     complex(4, +0)];
[apOut, w, z, info] = f08gn(jobz, uplo, n, ap)
```

```
apOut =
  -0.0049
   0.1559
  -1.6564
   0.3559 + 0.1524i
  -3.9039
   4.6613
   0.3367 + 0.0008i
   0.3567 - 0.0783i
  -7.8740
   4.0000
w =
   -5.6248
   -0.2182
    0.0638
   12.7793
z =
   1.0e-53 *
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3406 + 0.0000i        0
info =
           0
```